

Breaking RSA

The weak point of the RSA algorithm lies in the key generation: specifically, the difficulty of factoring (N) into its prime components. This factorisation is only possible through trial and error, and the algorithm remains secure until someone discovers a heuristic method to do it efficiently.

You wouldn't believe how tricky it is to find (p) and (q) when $(N = 77)$. Once you've figured it out, think about how you did it. What was the process? How did you come up with the solution?

The RSA Labs used to offer \$200,000 for the factorization of 2048-bit numbers. However, they removed the prize because more recent research has shown that the method can be broken if the primes (p) and (q) have a special relationship.

The RSA Labs previously offered a \$100,000 prize to anyone who could submit the prime factors of the following integer (N) , that is, the two primes whose product is (N) :

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N = p x q = 25195908475657893494027183240048398571429282126204
03202777713783604366202070759555626401852588078440
69182906412495150821892985591491761845028084891200
72844992687392807287776735971418347270261896375014
97182469116507761337985909570009733045974880842840
17974291006424586918171951187461215151726546322822
16869987549182422433637259085141865462043576798423
38718477444792073993423658482382428119816381501067
48104516603773060562016196762561338441436038339044
14952634432190114657544454178424020924616515723350
77870774981712577246796292638635637328991215483143
81678998850404453640235273819513786365643912120103
97122822120720357
```

You can sense how difficult this task is: we are dealing with 617 digits, and unfortunately, the last digit is not even.

Example: Breaking RSA if we find (p) and (q)

Step 1: Given Data

Suppose we know the **public key** (e, N) :

- Public exponent $(e = 17)$
- Modulus $(N = 55)$

To break the RSA encryption, we need to find the **private key** (d) . This requires us to factor (N) into its prime factors (p) and (q) .

Step 2: Factor (N) We need to factor $(N = 55)$:

- $(p = 5)$
- $(q = 11)$

These are the two prime factors of (N) .

Step 3: Calculate $(\phi(N))$ (Euler's Totient Function)

Once we have (p) and (q) , we can compute $(\phi(N))$, which is required to compute the private key (d) .

$$\phi(N) = (p - 1) \times (q - 1) \quad \phi(55) = (5 - 1) \times (11 - 1) = 4 \times 10 = 40$$

Step 4: Find the Private Key (d)

The private key (d) is the modular inverse of $(e \pmod{\phi(N)})$, which satisfies the equation:

$$d \times e \equiv 1 \pmod{\phi(N)}$$

We need to find (d) such that:

$$d \times 17 \equiv 1 \pmod{40}$$

Using the extended Euclidean algorithm, we find that:

$$d = 33$$

So the **private key** is $(d = 33)$.

Step 5: Decrypt the Ciphertext

Now that we have (d) , we can decrypt the ciphertext. Suppose the ciphertext is $(C = 18)$.

To decrypt, we use the formula:

$$m = C^d \pmod{N} \quad m = 18^{33} \pmod{55}$$

To compute this efficiently, we can use modular exponentiation:

$$18^{33} \pmod{55} = 2$$

So, the original **message** $(m = 2)$.

Summary:

- We started with the public key $(e = 17, N = 55)$ and a ciphertext $(C = 18)$.
- After factoring (N) into $(p = 5)$ and $(q = 11)$, we calculated $(\phi(N) = 40)$ and found the private key $(d = 33)$.
- Using (d) , we decrypted the ciphertext $(C = 18)$ to recover the original message $(m = 2)$.

This example illustrates how RSA encryption can be broken if someone manages to factor (N) into its prime factors (p) and (q) .

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