Conditional probability

How can we calculate the result in a case where two events are not independent. It means that, if one event occurs it will directly affect the probability for the other event?

If event A and B are those kind of complex events which will not exclude each other. In this case we have a so-called conditional probability (event A affects event B).

Notation: \(p(A | B) \)

In this case we mean the relative frequency which compares the sum of all probability to the probability of event B (probability of it's occurrance).

$$ p(A|B) = \frac{k_{AB}}{k_b} = \frac{\frac{k_{AB}}{k}}{\frac{k_{B}}{k_b}} = \frac{p(A \cap B)}{p(B)} $$

So we can get to the conclusion:

$$ p(A \cap B) = p(A|B) p(B) $$

1.) \(p(A \cap B) \): This represents the probability that both events A and B occur simultaneously. It is also known as the probability of the intersection of A and B.

2.) \(p(A|B) \): This is the conditional probability of event A occurring given that event B has already occurred. It tells us how likely A s to happen under the condition that B has happened.

What the Formula Says?

The formula states that the probability of both events A and B occurring together, is equal to the probability of B occurring multipliend by the probability of A occurring given that B has already occurred.

Example:

A manufacturer needs to produce a shaft with two critical dimensions: length (L) and diameter (D). Tolerances of \(L \pm \Delta \) and \(D \pm \Delta \) is allowed. After inspecting 180 components, the results are as follows:

Measurement Result	**Quantity**
Faultless \((H)\)	162
The length □ is faulty \((A)\)	10
The diameter □ is faulty \((B)\)	12
Both dimensions are faulty \((A \cap B) \)	4
Only the length □ is faulty \((C)\)	6
Only the diameter □ is faulty \(D)\)	8

Question: What are the probabilities of events \(A\) and \(B\):