

Conditional probability

How can we calculate the result in a case where two events are not independent. It means that, if one event occurs it will directly affect the probability for the other event?

If event A and B are those kind of complex events which will not exclude each other. In this case we have a so-called conditional probability (event A affects event B).

Notation: $(p(A | B))$

In this case we mean the relative frequency which compares the sum of all probability to the probability of event B (probability of its occurrence).

$$p(A|B) = \frac{k_{AB}}{k_B} = \frac{\frac{k_{AB}}{k}}{\frac{k_B}{k}} = \frac{p(A \cap B)}{p(B)}$$

So we can get to the conclusion:

$$p(A \cap B) = p(A|B) p(B)$$

1.) $(p(A \cap B))$: This represents the probability that both events A and B occur simultaneously. It is also known as the probability of the intersection of A and B.

2.) $(p(A|B))$: This is the conditional probability of event A occurring given that event B has already occurred. It tells us how likely A is to happen under the condition that B has happened.

What the Formula Says?

The formula states that the probability of both events A and B occurring together, is equal to the probability of B occurring multiplied by the probability of A occurring given that B has already occurred.

Example:

A manufacturer needs to produce a shaft with two critical dimensions: length (L) and diameter (D). Tolerances of $(L \pm \Delta)$ and $(D \pm \Delta)$ is allowed. After inspecting 180 components, the results are as follows:

| Measurement Result | Quantity |
|---|----------|
| Faultless $((H))$ | 162 |
| The length \square is faulty $((A))$ | 10 |
| The diameter \square is faulty $((B))$ | 12 |
| Both dimensions are faulty $((A \cap B))$ | 4 |
| Only the length \square is faulty $((C))$ | 6 |
| Only the diameter \square is faulty $((D))$ | 8 |

Question: What are the probabilities of events $((A))$ and $((B))$, and $(P(A | B))$? The probability of the event "length" is faulty $((A))$ is:

$$p(A) = \frac{10}{180} = 0.05555$$

The probability of the event "diameter" is faulty $((B))$ is:

$$p(B) = \frac{12}{180} = 0.06666$$

What is the probability that both dimensions are faulty?

$$p(A \cap B) = \frac{4}{180} = 0.0222$$

What is the probability that a shaft's length is faulty, given that its diameter is already faulty?

The conditional probability of both events occurring can be calculated using the definition:

$$p(A \mid B) = \frac{\text{\textit{both dimensions are faulty}}}{\text{\textit{diameter is faulty}}} = \frac{4}{12} = 0.3333$$

Since this does not match with the product $(p(A) p(B))$, we can conclude that the two events are **not independent!**

Thus, the joint probability can also be calculated differently:

$$p(A \cap B) = p(A \mid B) \cdot p(B) = 0.333 \cdot 0.0666 = 0.02222$$

The probability of event (C) is:

$$p(C) = \frac{6}{180} = 0.0333$$

The probability of event (D) is:

$$p(D) = \frac{8}{180} = 0.0444$$

The probability of defective production is:

$$p(H) = \frac{180 - 162}{180} = \frac{18}{180} = 0.1$$

Alternatively, we can calculate it as:

$$p(A \cup B) = p(A) + p(B) - p(A \cap B) = 0.0555 + 0.0666 - 0.0222 = 0.1$$

or

$$p(A \cup B \cup E) = 0.0333 + 0.0444 + 0.0222 = 0.1$$

where $(E = A \cap B)$

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