

Conditional probability

Non-independent events

How can we calculate the result when two events are not independent? If one event occurs, it will directly affect the probability of the other event.

If events **A** and **B** are complex events that will not exclude each other, we have a so-called conditional probability (event A affects event B).

Notation: $p(A | B)$

In this case, we mean the relative frequency, which compares the sum of all probabilities to the probability of event B (the probability of its occurrence).

$$p(A|B) = \frac{k_{AB}}{k_B} = \frac{\frac{k_{AB}}{k}}{\frac{k_B}{k}} = \frac{p(A \cap B)}{p(B)}$$

So we can get to the conclusion:

$$p(A \cap B) = p(A|B) p(B)$$

1.) $p(A \cap B)$: This represents the probability that both events A and B occur simultaneously, which is also known as the probability of their intersection.

2.) $p(A|B)$: This is the conditional probability of event A occurring given that event B has already occurred. It tells us how likely A is to happen under the condition that B has happened.

What the Formula Says?

The formula states that the probability of events **A** and **B** occurring together, is equal to the probability of B occurring multiplied by the probability of A occurring given that B has already occurred.

Example:

A manufacturer must produce a shaft with two critical dimensions: length (L) and diameter (D). Tolerances of $L \pm \Delta$ and $D \pm \Delta$ are allowed. After inspecting 180 components, the results are as follows:

Measurement Result	Quantity
Faultless ((H))	162
The length (A) is faulty ((A))	10
The diameter (B) is faulty ((B))	12
Both dimensions are faulty ((A ∩ B))	4
Only the length (A) is faulty ((C))	6
Only the diameter (B) is faulty ((D))	8

Question 1: What are the probabilities of events (A) and (B) ?

The probability of the event “length” is faulty “ (A) ” is:

$$\$ \$ p(A) = \frac{10}{180} = 0.05555 \$ \$$$

The probability of the event “diameter” is faulty” $\cap (B)$ is:

$$\$ \$ p(B) = \frac{12}{180} = 0.06666 \$ \$$$

Question 2: What is the probability that both dimensions are faulty?

$$\$ \$ p(A \cap B) = \frac{4}{180} = 0.0222 \$ \$$$

Question 3: What is the probability that a shaft's length is faulty, given that its diameter is already faulty?

The conditional probability of both events occurring can be calculated using the definition:

$$\$ \$ p(A \mid B) = \frac{\text{both dimensions are faulty}}{\text{diameter is faulty}} = \frac{4}{12} = 0.3333 \$ \$$$

Since this does not match with the product $p(A) p(B)$, we can conclude that the two events are **not independent!**

Thus, the joint probability can also be calculated differently:

$$\$ \$ p(A \cap B) = p(A \mid B) \cdot p(B) = 0.333 \cdot 0.0666 = 0.02222 \$ \$$$

The probability of event $\cap (C)$ is:

$$\$ \$ p(C) = \frac{6}{180} = 0.0333 \$ \$$$

The probability of event $\cap (D)$ is:

$$\$ \$ p(D) = \frac{8}{180} = 0.0444 \$ \$$$

Question 4: What is the probability of defective production?

$$\$ \$ p(H) = \frac{180 - 162}{180} = \frac{18}{180} = 0.1 \$ \$$$

Alternatively, we can calculate it as:

$$\$ \$ p(A \cup B) = p(A) + p(B) - p(A \cap B) = 0.0555 + 0.0666 - 0.0222 = 0.1 \$ \$$$

or

$$\$ \$ p(A \cup B \cup E) = 0.0333 + 0.0444 + 0.0222 = 0.1 \$ \$$$

where $\cap (E = A \cap B)$

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