2025/11/03 17:34 1/3 Conditional probability

Conditional probability

Non-independent events

How can we calculate the result when two events are not independent? If one event occurs, it will directly affect the probability of the other event.

If events **A** and **B** are complex events that will not exclude each other, we have a so-called conditional probability (event A affects event B).

Notation: \(p(A | B) \)

In this case, we mean the relative frequency, which compares the sum of all probabilities to the probability of event B (the probability of its occurrence).

$$p(A|B) = \frac{k_{AB}}{k_b} = \frac{k_{AB}}{k} = \frac{k_{B}}{k} = \frac{p(A \land B)}{p(B)}$$

So we can get to the conclusion:

$$$$ p(A \subset B) = p(A|B) p(B) $$$$

- 1.) \(p(A \cap B) \): This represents the probability that both events A and B occur simultaneously, which is also known as the probability of their intersection.
- 2.) (p(A|B)): This is the conditional probability of event A occurring given that event B has already occurred. It tells us how likely A s to happen under the condition that B has happened.

What the Formula Says?

The formula states that the probability of events **A** and **B** occurring together, is equal to the probability of B occurring multiplied by the probability of A occurring given that B has already occurred.

Example 1:

A manufacturer must produce a shaft with two critical dimensions: length (L) and diameter (D). Tolerances of \(L \pm \Delta \) and \(D \pm \Delta \) is allowed. After inspecting 180 components, the results are as follows:

Measurement Result	Quantity
Faultless \((H)\)	162
The length □ is faulty \((A)\)	10
The diameter □ is faulty \((B)\)	12
Both dimensions are faulty \((A \cap B) \)	4
Only the length ☐ is faulty \((C)\)	6
Only the diameter ☐ is faulty \(D)\)	8

Question 1: What are the probabilities of events \(A\) and \(B\)?

The probability of the event "length" is faulty" \((A)\) is:

$$p(A) = \frac{10}{180} = 0.05555$$

The probability of the event "diameter" is faulty" \((B)\) is:

$$p(B) = \frac{12}{180} = 0.06666$$
\$

Question 2: What is the probability that both dimensions are faulty?

$$p(A \subset B) = \frac{4}{180} = 0.0222$$
\$\$

Question 3: What is the probability that a shaft's length is faulty, given that its diameter is already faulty?

The conditional probability of both events occurring can be calculated using the definition:

\$\$ p(A \mid B) = \frac{\text{both dimensions are faulty}}{\text{diameter is faulty}} = \frac{4}{12} = 0.3333 \$\$

Since this does not match with the product (p(A) p(B)), we can conclude that the two events are **not** independent!

Thus, the joint probability can also be calculated differently:

$$p(A \subset B) = p(A \subset B) \setminus (A \subset B) = 0.333 \setminus (A \subset B) = 0.02222$$

The probability of event \(C\) is:

$$p(C) = \frac{6}{180} = 0.0333$$
\$\$

The probability of event \(D\) is:

$$p(D) = \frac{8}{180} = 0.0444$$

Question 4: What is the probability of defective production?

$$p(H) = \frac{180 - 162}{180} = \frac{18}{180} = 0.1$$
\$

Alternatively, we can calculate it as:

$$p(A \setminus B) = p(A) + p(B) - p(A \setminus B) = 0.0555 + 0.0666 - 0.0222 = 0.1$$
\$

or

$$p(A \subset B \subset E) = 0.0333 + 0.0444 + 0.0222 = 0.1$$
\$\$

where
$$\ (E = A \setminus B)$$

Example 2: Find the conditional probability of a machine breakdown given that preventive maintenance was performed.

We have the following information:

The probability that the machine breaks down (Event A) is \(p(A) = 0.10 \)

2025/11/03 17:34 3/3 Conditional probability

• The probability that the machine undergoes preventive maintenance (Event B) is (p(B) = 0.30)

 The probability that the machine both breaks down and has undergone preventive maintenance is \(p(A \cap B) = 0.015 \)

To calculate the conditional probability of the machine breaking down given that it underwent preventive maintenance, we apply the conditional probability formula:

$$p(A|B) = \frac{p(A \land B)}{p(B)}$$

Substitute the values:

$$p(A|B) = \frac{0.015}{0.30}$$
\$\$

Thus, the probability that the machine breaks down given that it underwent preventive maintenance is (p(A|B) = 0.05), or 5%.

From:

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Last update: 2024/09/30 12:30

