Conditional probability

Non-independent events

How can we calculate the result when two events are not independent? If one event occurs, it will directly affect the probability of the other event.

If events **A** and **B** are complex events that will not exclude each other, we have a so-called conditional probability (event A affects event B).

Notation: $p(A | B)$

In this case, we mean the relative frequency, which compares the sum of all probabilities to the probability of event B (the probability of its occurrence).

$$ p(A|B) = \frac{k_{AB}}{k_b} = \frac{\frac{k_{AB}}{k}}{\frac{k_{B}}{k}} = \frac{p(A \cap B)}{p(B)} $$

So we can get to the conclusion:

$$ p(A \cap B) = p(A|B) p(B) $$

1.) $p(A \cap B)$: This represents the probability that both events A and B occur simultaneously, which is also known as the probability of their intersection.

2.) $p(A|B)$: This is the conditional probability of event A occurring given that event B has already occurred. It tells us how likely A s to happen under the condition that B has happened.

What the Formula Says?

The formula states that the probability of events **A** and **B** occurring together, is equal to the probability of B occurring multiplied by the probability of A occurring given that B has already occurred.

Example 1:

A manufacturer must produce a shaft with two critical dimensions: length (L) and diameter (D). Tolerances of $L \pm \Delta$ and $D \pm \Delta$ is allowed. After inspecting 180 components, the results are as follows:

Measurement Result	**Quantity**
Faultless (H)	162
The length □ is faulty (A)	10
The diameter □ is faulty (B)	12
Both dimensions are faulty $(A \cap B)$	4
Only the length □ is faulty (C)	6
Only the diameter □ is faulty D	8

Question 1: What are the probabilities of events A and B?

The probability of the event “length” is faulty“ (A) is:

$$ p(A) = \frac{10}{180} = 0.05555 $$

The probability of the event “diameter” is faulty” (B) is:

$$ p(B) = \frac{12}{180} = 0.06666 $$

Question 2: What is the probability that both dimensions are faulty?

$$ p(A \cap B) = \frac{4}{180} = 0.0222 $$

Question 3: What is the probability that a shaft's length is faulty, given that its diameter is already faulty?

The conditional probability of both events occurring can be calculated using the definition:

$$ p(A \mid B) = \frac{\text{both dimensions are faulty}}{\text{diameter is faulty}} = \frac{4}{12} = 0.3333 $$

Since this does not match with the product $p(A) \cdot p(B)$, we can conclude that the two events are **not independent**!

Thus, the joint probability can also be calculated differently:

$$ p(A \cap B) = p(A \mid B) \cdot p(B) = 0.333 \cdot 0.0666 = 0.02222 $$

The probability of event C is:

$$ p(C) = \frac{6}{180} = 0.0333 $$

The probability of event D is:

$$ p(D) = \frac{8}{180} = 0.0444 $$

Question 4: What is the probability of defective production?

$$ p(H) = \frac{180 - 162}{180} = \frac{18}{180} = 0.1 $$

Alternatively, we can calculate it as:

$$ p(A \cup B) = p(A) + p(B) - p(A \cap B) = 0.0555 + 0.0666 - 0.0222 = 0.1 $$

or

$$ p(A \cup B \cup E) = 0.0333 + 0.0444 + 0.0222 = 0.1 $$

where $E = A \cap B$

Example 2: Find the conditional probability of a machine breakdown given that preventive maintenance was performed.

We have the following information:

- The probability that the machine breaks down (Event A) is $p(A) = 0.10$

- The probability that the machine undergoes preventive maintenance (Event B) is $p(B) = 0.30$
- The probability that the machine both breaks down and has undergone preventive maintenance is $p(A \cap B) = 0.015$

To calculate the conditional probability of the machine breaking down given that it underwent preventive maintenance, we apply the conditional probability formula:

$$ p(A|B) = \frac{p(A \cap B)}{p(B)} $$

Substitute the values:

$$ p(A|B) = \frac{0.015}{0.30} $$

Thus, the probability that the machine breaks down given that it underwent preventive maintenance is $p(A|B) = 0.05$, or 5%.